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# Activated Sludge Calculations with Excel

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Harlan H. Bengtson, PhD, P.E.

## COURSE CONTENT

### 1. Introduction

The activated sludge process is very widely used for biological wastewater treatment. This course includes background on biological wastewater treatment, a general description of the activated sludge process, information about several variations of the activated sludge process, discussion of design calculations for an activated sludge aeration tank, and discussion of activated sludge operational calculations. Example calculations and examples illustrating the setup and use of an Excel spreadsheet for making activated sludge design and operational calculations are also included.



Image Credit:

[Wikimedia Commons – Beckton STP, Activated Sludge Tank – geography.org.uk](#)

## 2. Learning Objectives

At the conclusion of this course, the student will

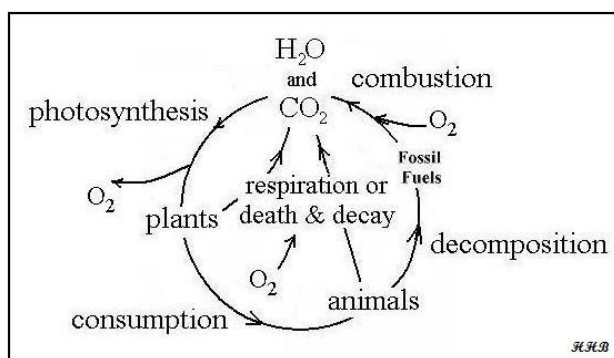
- Know the equation for biological oxidation and how it fits into the organic carbon cycle
- Know the equation for photosynthesis and how it fits into the organic carbon cycle
- Know the major components of an activated sludge wastewater treatment system
- Be able to describe the differences between an extended aeration and conventional activated sludge system
- Be able to describe the differences between a contact stabilization and conventional activated sludge system
- Be able to calculate required aeration tank volume (in U.S. units) for a specified volumetric loading, hydraulic residence time, or aeration tank F:M ratio, if given suitable aeration tank influent and aeration tank parameter information
- Be able to calculate required aeration tank volume (in S.I. units) for a specified volumetric loading, hydraulic residence time, or aeration tank F:M ratio, if given suitable aeration tank influent and aeration tank parameter information
- Be able to calculate the required activated sludge recycle flow rate, waste activated sludge flow rate, and aeration tank F:M ratio, if given suitable wastewater stream and aeration tank information along with the desired value for sludge retention time
- Be able to use an Excel spreadsheet for making activated sludge design and operational calculations

### 3. Topics Covered in this Course

- I. Biochemical Oxygen Demand as a cause of Water Pollution
- II. Activated Sludge Background
- III. Activated Sludge Process Variations
- IV. Activated Sludge Parameters
- V. Activated Sludge Design Calculations
- VI. Activated Sludge Operational Calculations
- VII. Explanation of Equations for  $Q_w$  and  $Q_r$
- VIII. Summary
- IX. References

### 4. Biochemical Oxygen Demand as a Cause of Water Pollution

Biochemical oxygen demand (BOD) is an indirect measure of the concentration of biodegradable organic matter in water or wastewater. Organic matter (as measured by BOD) is one of the major constituents removed from wastewater in domestic wastewater treatment plants. The



The Organic Carbon Cycle

carbon cycle shown in the above diagram is the portion that takes place in the biological treatment component of a wastewater treatment plant or else

takes place in the receiving stream if the organic matter isn't removed in the treatment plant.

The two major reactions that take place in the organic carbon cycle are biological oxidation of waste organic matter and photosynthesis, which is the process by which green plants produce organic matter from carbon dioxide and water in reactions that are catalyzed by sunlight and the chlorophyll in the green plants. Through the biological oxidation process, aerobic microorganisms utilize oxygen in breaking down organic matter to carbon dioxide and water together with small amounts of other end products.

The photosynthesis and biological oxidation processes can be represented by the following two equations:

Photosynthesis:

**CO<sub>2</sub> + H<sub>2</sub>O + sunlight → organic plant matter** (primarily C, H, & O) + **oxygen** (this reaction is catalyzed by the chlorophyll in green plants)

Biological Oxidation:

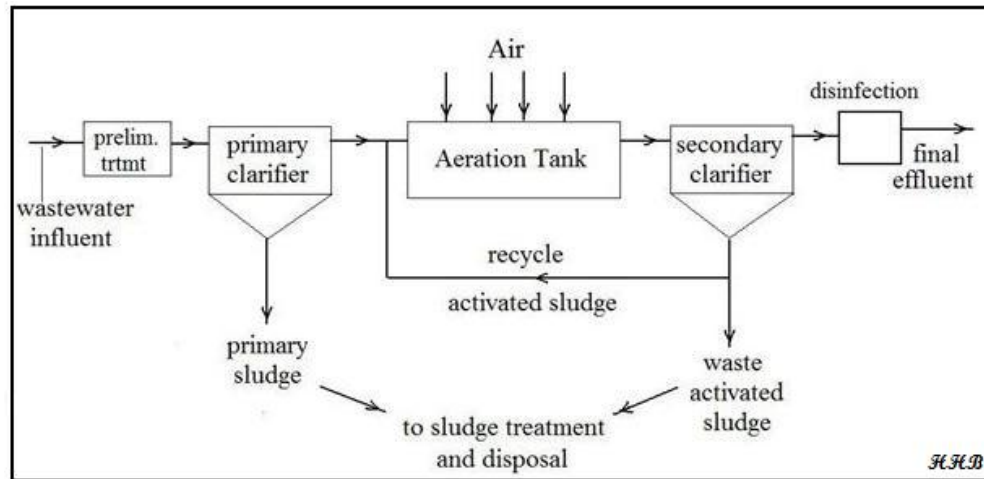
**Waste organic matter** (primarily C, H & O) + **O<sub>2</sub>** → **CO<sub>2</sub> + H<sub>2</sub>O + energy**

This reaction is the 'death and decay' shown in the organic carbon cycle diagram. The process takes place as aerobic microorganisms utilize the waste organic matter as their food (energy) source. The process uses oxygen, so if it is taking place in a water body, dissolved oxygen is consumed. A large quantity of organic matter in the water will result in multiplication of microorganisms and rapid removal of dissolved oxygen, leading to oxygen depletion below the level needed by aquatic life.

This is also the process that takes place in biological oxidation processes in wastewater treatment plants for removal of organic matter from the incoming wastewater.

## 5. Activated Sludge Background

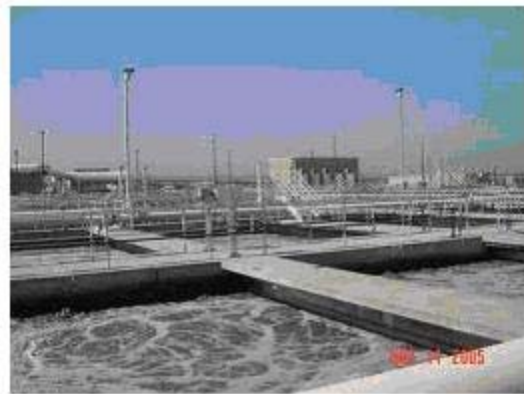
The diagram below shows a general flow diagram with the typical components present in an activated sludge wastewater treatment plant.



Activated Sludge Wastewater Treatment Flow Diagram

The first component is preliminary treatment, typically consisting of screening, flow measurement, and perhaps grit removal. The second component, the primary clarifier, is used to remove settleable suspended matter. The underflow goes to sludge treatment and disposal and the overflow goes to an aeration tank.

The aeration tank is the heart of an activated sludge treatment process. It is here that biological oxidation of dissolved and fine suspended organic matter takes place. The biological oxidation takes place because aerobic microorganisms, organic matter and dissolved oxygen are all brought together in the aeration tank. The organic matter comes in with the primary effluent. The dissolved oxygen level is maintained by blowing air into the aeration tank through diffusers (or in some cases with a



Activated Sludge Aeration Tanks

mechanical aerator). This also serves to keep the aeration tank contents mixed. A suitable concentration of microorganisms is maintained in the aeration tank by settling out the ‘activated sludge’ (microorganisms) in the secondary clarifier and recycling them back into the aeration tank. (Image Credit: Lee & Ro, Inc)

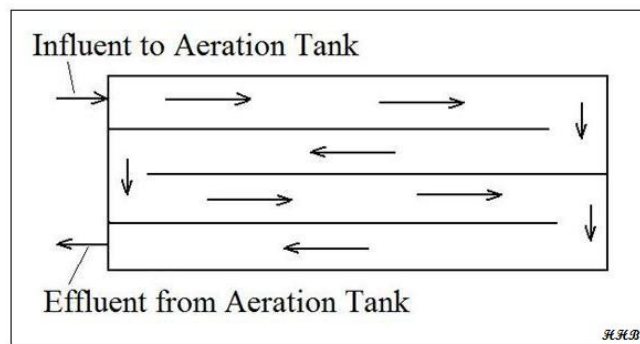
## 6. Activated Sludge Process Variations

Four common variations of the activated sludge process are:

- Conventional activated sludge
- Extended aeration
- Completely mixed activated sludge
- The contact stabilization process

A brief description of each follows.

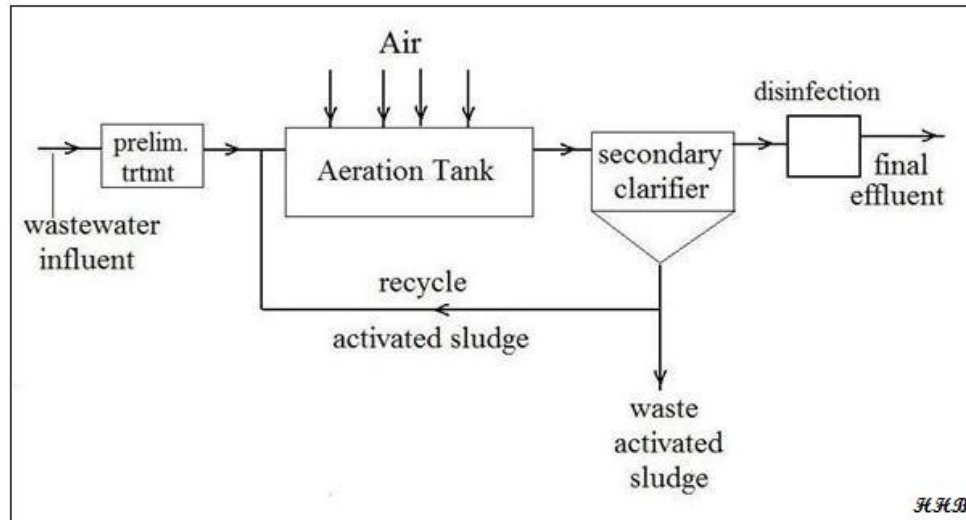
The Conventional Activated Sludge Process is used over a wide range of wastewater flow rates, from small to very large plants. The flow diagram and general description is that given above in the Activated Sludge Background section. The aeration tank in a conventional activated sludge process is typically designed with a long, narrow configuration, thus giving approximately ‘plug flow’ through the tank. For large treatment plants, the aeration tank is often built with a serpentine pattern, like that shown in the diagram at the right, in order to obtain the desired plug flow without an excessive length requirement for the tank.



Serpentine Pattern Activated Sludge Aeration Tank

The Extended Aeration Activated Sludge Process is shown in the diagram below. As you can see, this process doesn’t use a primary clarifier. Instead, a longer detention time is used for the aeration tank, so that the settleable organic matter will be biologically oxidized along with the dissolved and

fine suspended organic matter. This requires a hydraulic detention time of about 24 hours instead of the 6 to 8 hours that is typical for the conventional activated sludge process. This simplifies the operation of the plant by eliminating the primary clarifier and reducing the need for sludge treatment and disposal to a very minimal flow of waste activated sludge that must be drawn off periodically.

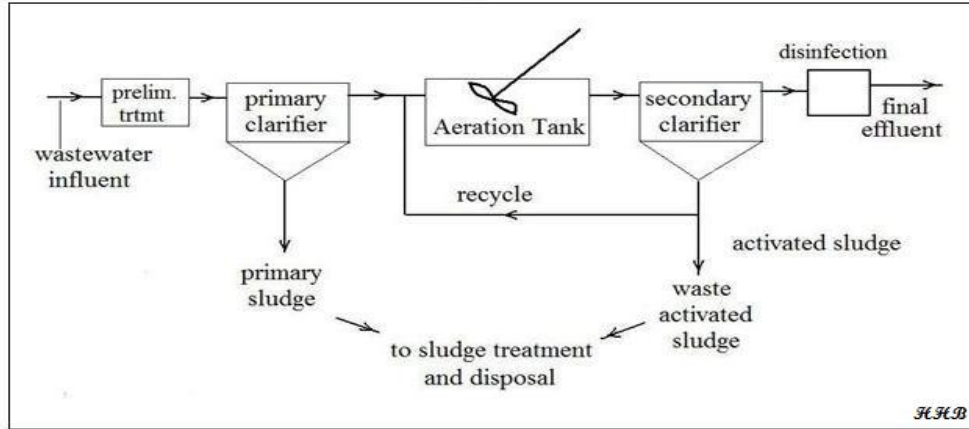


Extended Aeration Activated Sludge Wastewater Treatment Process

The Completely Mixed Activated Sludge Process has the same overall flow pattern as the conventional activated sludge process. The main differences are the method of aeration and the aeration tank configuration. For the completely mixed option, aeration is usually with a mechanical mixer, rather than with diffused air. Also, the tank configuration is usually approximately square, rather than long and narrow. This combination of mixing and tank configurations makes the aeration tank approximate a completely mixed reactor rather than the plug flow reactor approximated by the conventional activated sludge aeration tank. The flow diagram below illustrates this.

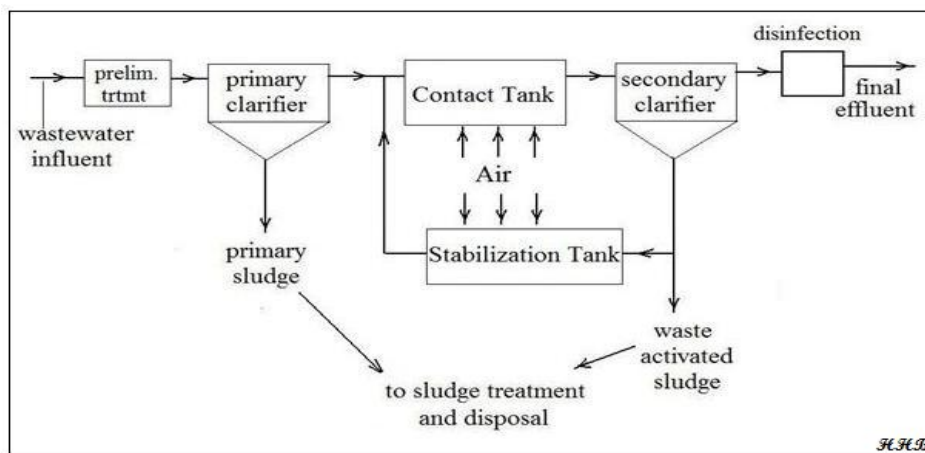
Typical applications of the completely mixed activated sludge process are cases where slug flows of high concentration, hard to oxidize, or toxic wastes enter the treatment plant. The complete mixing dilutes such flows into the entire tank contents more rapidly than a plug flow design, making the slug flow less likely to upset or kill the microorganisms.





Completely Mixed Activated Sludge Flow Diagram

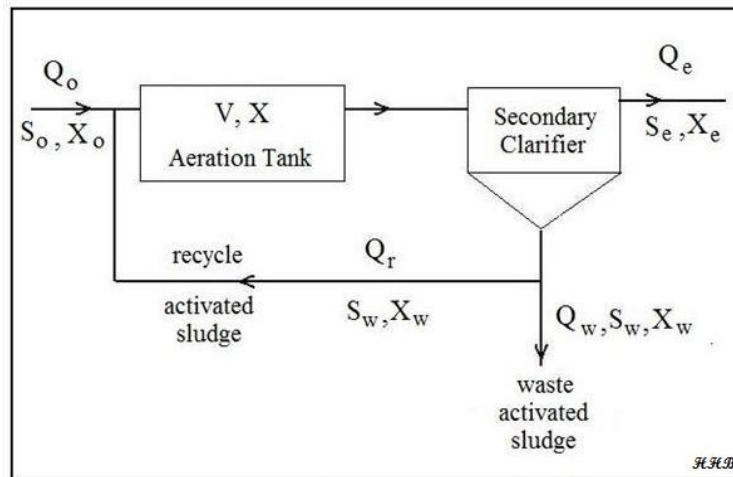
The Contact Stabilization Activated Sludge Process gets by with less total aeration tank volume than that needed for the conventional activated sludge process. This is accomplished because the full wastewater flow is aerated for only 0.5 to 2 hours in an aerated contact tank. This is sufficient time for removal of the organic matter from the wastewater flow by the microorganism. If those microorganisms were recycled directly into the aeration tank after settling out in the secondary clarifier, however, they would not continue to take up organic matter, because they are “still full” from the 0.5 to 2 hour feast they recently had. If the recycle sludge is aerated for 3 to 8 hours to allow the microorganisms to “digest” the organic matter that they’ve taken up, then they go back into the aeration tank ready to go to work. Since the recycle activated sludge flow is less than the full wastewater flow, this results in less overall aeration tank volume for a given wastewater flow rate to be treated. The diagram below shows this process.



Contact Stabilization Activated Sludge Flow Diagram

## 7. Activated Sludge Parameters

The diagram below shows an activated sludge aeration tank and secondary clarifier with parameters for the primary effluent, secondary effluent, waste activated sludge, and recycle activated sludge streams.



Activated Sludge Flow Diagram & Parameters

The parameters in the diagram and a few others that will be used for the upcoming activated sludge calculations are summarized in the list below.

- primary effluent flow rate,  $Q_o$ , MGD ( $m^3/\text{day}$  for S.I.)
- primary effluent biochemical oxygen demand (BOD)
- concentration,  $S_o$ , mg/L ( $g/m^3$  for S.I.)
- primary effluent suspended solids conc.,  $X_o$ , mg/L ( $g/m^3$  for S.I.)
- aeration tank volume,  $V$ ,  $ft^3$  ( $m^3$  for S.I.)
- aeration tank MLSS (suspended solids conc.),  $X$ , mg/L ( $g/m^3$  for S.I.)
- secondary effluent flow rate,  $Q_e$ , MGD, ( $m^3/\text{day}$  for S.I.)
- secondary effluent susp. solids conc.,  $X_e$ , mg/L ( $g/m^3$  for S.I.)

- secondary effluent biochemical oxygen demand (BOD) concentration,  $S_e$ , mg/L ( $\text{g/m}^3$  for S.I.)
- waste activated sludge flow rate,  $Q_w$ , MGD ( $\text{m}^3/\text{day}$  for S.I.)
- waste activated sludge biochemical oxygen demand (BOD) conc.,  $S_w$ , mg/L ( $\text{g/m}^3$  for S.I.)
- waste activated sludge susp. solids conc.,  $X_w$  mg/L ( $\text{g/m}^3$  for S.I.)
- recycle activated sludge flow rate,  $Q_r$ , MGD ( $\text{m}^3/\text{day}$  for S.I.)
- Food to Microorganism ratio, **F:M**, lb BOD/day/lb MLVSS (kg BOD/day/kg MLVSS)
- Hydraulic retention time, **HRT**, hours (hours for S.I.)
- Sludge retention time (also called sludge age), **SRT**, days (days for S.I.)
- Volumetric loading, **VL**, lb BOD/day/1000  $\text{ft}^3$  (kg BOD/day/ $\text{m}^3$  for S.I.)
- % volatile solids in the aeration tank mixed liquor suspended solids, **%Vol.**

## 8. Activated Sludge Design Calculations

The table below shows typical values for three design parameters sometimes used for sizing activated sludge aeration basins: volumetric loading, food to microorganism ratio (F:M), and hydraulic residence time (HRT). Note that values for volumetric loading are given in both U.S. and S.I. units. The other two parameters are the same for either U.S. or S.I. units, since F:M will be the same expresses as either lb BOD/day/lb MLVSS or kg BOD/day/kg MLVSS, and HRT is simply in hours.

## Activated Sludge Design Parameters - Typical Ranges

Activated Sludge Process	Volumetric Loading		F:M	HRT hours
	lb BOD/day 1000 ft <sup>3</sup>	kg BOD/day m <sup>3</sup>	kg BOD/day kg MLVSS	
Conventional Plug Flow	20 -40	0.3 - 0.7	0.2 - 0.4	4 - 8
Complete Mix	20 - 100	0.3 - 1.6	0.2 - 0.6	3 - 5
Extended Aeration	5 - 15	0.1 - 0.3	0.04 - 0.1	20 - 30

(adapted from Metcalf & Eddy - Reference #1)

**NOTE:** F:M values will be the same for units of lb BOD/day/lb MLVSS.

Calculations with these design parameters can be made in U.S. units using the following equations:

- $V = [ (8.34 * S_o * Q_o) / VL ] (1000)$
- $V_{MG} = V * 7.48 / 1,000,000$
- $HRT = 24 * V_{MG} / Q_o$
- $F:M = (8.34 * S_o * Q_o) / (8.34 \% Vol * X * V_{MG})$   
 $= (S_o * Q_o) / (\% Vol * X * V_{MG})$

$V_{MG}$  is the tank volume in millions of gallons. It is introduced for convenience in calculations, since the primary effluent flow rate is given in MGD. The other parameters in the equations are as defined in the list above. The 8.34 factor in the equations above is used to convert mg/L to lb/MG, and the 7.48 is for conversion of ft<sup>3</sup> to gallons. Also, note that the primary sludge flow rate is typically very small in comparison with the influent wastewater flow rate, so the primary effluent flow rate,  $Q_o$ , is typically taken to be equal to the plant influent flow rate.

**Example #1:** Calculate the aeration tank volume requirement for a conventional activated sludge plant treating a daily average flow rate of 3.5 MGD, with primary effluent BOD estimated to be 175 mg/L. The design criterion is to be a volumetric loading rate of 30 lb BOD/day/1000 ft<sup>3</sup>.

**Solution:** The required volume can be calculated from the first equation in the list above:

$$V = [ (8.34 * S_o * Q_o) / VL ] (1000) = 8.34 * 175 * 3.5 * 1000 / 30 = \underline{170,275 \text{ ft}^3}$$

**Example #2:** For an assumed aeration tank MLSS of 2100 mg/L and assumed % volatile MLSS of 75%, what would be the aeration tank F:M ratio and hydraulic residence time for the plant inflow and aeration tank volume from Example #1?

**Solution:** The HRT and F:M ratio can be calculated using the last three equations in the list above, as follows:

$$V_{MG} = V * 7.48 / 1,000,000 = 170,275 * 7.48 / 1,000,000 = 1.27 \text{ MG}$$

$$\text{HRT} = 24 * V_{MG} / Q_o = 24 * 1.27 / 3.5 = \underline{8.7 \text{ hours}}$$

$$\begin{aligned} \text{F:M} &= (8.34 * S_o * Q_o) / (8.34 * \%Vol * X * V_{MG}) = (175 * 3.5) / (0.75 * 2100 * 1.27) \\ &= \underline{0.31 \text{ lb BOD/day/lb MLVSS}} \end{aligned}$$

**Example #3:** Set up and use an Excel spreadsheet to find the solutions to Examples #1 and #2.

**Solution:** The screenshot on the next page shows an Excel spreadsheet with the solution to this example.

This spreadsheet is set up for user entry of specified design volumetric loading, VL, and input values for primary effluent flow rate, Q<sub>o</sub>, primary effluent biochemical oxygen demand (BOD), S<sub>o</sub>, aeration tank MLSS, X, and % volatile solids in the aeration tank, %Vol.

The spreadsheet then uses the equations presented above to calculate the design aeration tank volume and the resulting values for the other two design parameters, **F:M** and **HRT**.

To solve this Example problem, the given values (  $Q_o = 3.5$  MGD,  $S_o = 175$  mg/L,  $VL = 30$  lb BOD/day/1000 ft<sup>3</sup>,  $X = 2100$  mg/L, % Vol = 75% ) were entered into the blue cells in the upper left portion of the spreadsheet. The spreadsheet then calculated the parameters in the yellow cells in the upper right portion of the spreadsheet. As shown in the screenshot below, the same values for **V**, **HRT**, and **F:M** are obtained as those shown above for Example #1 and Example #2:

$$V = \underline{170,275 \text{ ft}^3}$$

$$\text{HRT} = \underline{8.7 \text{ hours}}$$

$$\text{F:M} = \underline{0.31 \text{ lb BOD/day/lb MLVSS}}$$

Activated Sludge Waste Water Treatment Calculations - U.S. units					
1. Aeration Tank Design					
Instructions: Enter values in blue boxes. Spreadsheet calculates values in yellow boxes					
Inputs			Calculations		
Prim. Effl. Flow Rate, $Q_o =$	3.5	MGD	( Design Based on Volumetric Loading )		
Prim. Effl. BOD, $S_o =$	175	mg/L	Aeration tank volume, $V =$	170,275	ft <sup>3</sup>
Aeration tank MLSS, $X =$	2100	mg/L	Aeration tank vol. $V_{MG} =$	1.27	MG
Design Vol. Loading, $VL =$	30	(lb BOD/day/1000 ft <sup>3</sup> )	Check on other design parameters:		
			Aeration tank <b>HRT</b> =	8.7	hr
% volatile MLSS, %Vol =	75%		Aeration tank <b>F:M</b> =	0.31	
			(lb BOD/day/lb MLVSS)		

Calculations in S.I. units can be made using the following equations:

- $V = (S_o * Q_o / 1000) / VL$
- $HRT = 24 * V / Q_o$
- $F:M = (S_o * Q_o) / (\%Vol * X * V)$

The equations are slightly simpler, because the S.I system doesn't have a strange volume unit like the gallon! The S.I. units for all of the parameters are given in the long list above. Note that the S.I. concentration unit g/m<sup>3</sup> is numerically equal to mg/L.

**Example #4:** Calculate the aeration tank volume requirement for a conventional activated sludge plant treating a daily average flow rate of 20,000 m<sup>3</sup>/day, with primary effluent BOD estimated to be 140 g/m<sup>3</sup>. The design criterion is to be a volumetric loading rate of 0.5 kg BOD/day/m<sup>3</sup>.

**Solution:** The required volume can be calculated from the first equation in the list above:

$$V = (S_o * Q_o / 1000) / VL = (140 * 20,000 / 1000) / 0.5 = \underline{5,600 \text{ m}^3}$$

**Example #5:** For an assumed aeration tank MLSS of 2100 mg/L and assumed % volatile MLSS of 75%, what would be the aeration tank F:M ratio and hydraulic residence time for the plant inflow and aeration tank volume from Example #3?

**Solution:** The HRT and F:M ratio can be calculated using the last two equations in the list above, as follows:

$$HRT = 24 * V / Q_o = 24 * 5,600 / 20,000 = \underline{6.7 \text{ hr}}$$

$$F:M = (S_o * Q_o) / (\%Vol * X * V) = (140 * 20,000) / (0.75 * 2100 * 5600) \\ = \underline{0.32 \text{ kg BOD/day/kg MLVSS}}$$

## 9. Activated Sludge Operational Calculations

The table below shows typical ranges for several operational activated sludge waste water treatment process parameters. Note that these values remain the same for U.S. or S.I. units. **SRT** will still have units of days for the U.S. or S.I. system. **MLSS** concentration will have the S.I. unit of  $\text{g/m}^3$ , which is numerically equal to  $\text{mg/L}$ . **F:M** will have the S.I. unit of  $\text{kg BOD/day/kg MLVSS}$ , which is numerically equal to  $\text{lb BOD/day/lb MLVSS}$ . The % unit for  $Q_r/Q_o$  remains the same for U.S. or S.I. units.

Activated Sludge Operational Parameters - Typical Ranges

Activated Sludge Process	SRT days	MLSS mg/L	F:M $\frac{\text{lb BOD/day}}{\text{lb MLVSS}}$	$Q_r/Q_o$ %
Conventional Plug Flow	3 - 15	1000 - 3000	0.2 - 0.4	25 - 75
Complete Mix	3 - 15	1500 - 4000	0.2 - 0.6	25 - 100
Extended Aeration	20 - 40	2000 - 5000	0.04 - 0.1	50 - 150

Calculations with these parameters can be made in U.S. units using the following equations:

- $Q_r = [Q_o(X - X_o) - Q_w X_w] / (X_w - X)$  [sometimes approximated by:  
 $Q_r = Q_o(X - X_o) / (X_w - X)$ ] (See discussion in next section of course)
- $V_{MG} = V * 7.48 / 1,000,000$
- $Q_w = (8.34 * V_{MG} * X) / (8.34 * \text{SRT} * X_w) = (V_{MG} * X) / (\text{SRT} * X_w)$
- $F:M = (8.34 * S_o * Q_o) / (8.34 \% \text{Vol} * X * V_{MG})$   
 $= (S_o * Q_o) / (\% \text{Vol} * X * V_{MG})$



The aeration tank volume in millions of gallons,  $V_{MG}$ , is used primarily in calculating  $Q_w$  and the **F:M** ratio.

**Example #6:** For the 3.5 MGD activated sludge plant in Example #1, with an aeration tank volume of 170,275 ft<sup>3</sup>, calculate a) the required recycle activated sludge flow rate (using the simplified equation), b) the waste activated sludge flow rate, and c) the aeration tank F:M ratio, based on the following: primary effluent BOD = 175 mg/L, primary effluent TSS = 200 mg/L, waste/recycle activated sludge SS concentration = 7,000 mg/L, aeration tank MLSS = 2000 mg/L, % volatile solids in the aeration tank = 75%, intended sludge retention time = 12 days.

**Solution:** a) The sludge recycle rate needed can be calculated from the simplified form of first equation in the list above:

$$Q_r = Q_o(X - X_o)/(X_w - X) = 3.5(2000 - 200)/(7000 - 2000) = \underline{\underline{1.3 \text{ MGD}}}$$

b) The waste activated sludge flow rate needed to give SRT = 12 days can be calculated from the second and third equations in the list above:

$$V_{MG} = V * 7.48 / 1,000,000 = 170,275 * 7.48 / 1,000,000 = 1.27 \text{ million gallons}$$

$$Q_w = (V_{MG} * X) / (SRT * X_w) = (1.27 * 2000) / (12 * 7000) = \underline{\underline{0.030 \text{ MGD}}}$$

or **30,000 gal/day**

c) The aeration tank F:M ratio can be calculated using the last equation in the list above together with the equation for  $V_{MG}$ :

$$V_{MG} = V * 7.48 / 1,000,000 = 170,275 * 7.48 / 1,000,000 = 1.27 \text{ million gallons}$$

$$F:M = (S_o * Q_o) / (\%Vol * X * V_{MG}) = (175 * 3.5) / (0.75 * 2000 * 1.27)$$

$$= \underline{\underline{0.32 \text{ lb BOD/day/lb MLVSS}}}$$

**Example #7:** Use an Excel spreadsheet to find the solution to Example #6.

**Solution:** The screenshot on the next page shows an Excel spreadsheet with the solution to this example.

This spreadsheet is set up for user entry of primary effluent flow rate,  $Q_o$ , primary effluent biochemical oxygen demand (BOD),  $S_o$ , primary effluent TSS,  $X_o$ , recycle/waste activated sludge concentration,  $X_w$ , aeration tank volume,  $V$ , aeration tank MLSS,  $X$ , % volatile solids in the aeration tank, %Vol, and the target value for sludge retention time, SRT. The spreadsheet then uses the equations presented above to calculate the recycle activated sludge flow rate,  $Q_r$ , the waste activated sludge flow rate,  $Q_w$ , and the aeration tank **F:M** ratio.

Activated Sludge Waste Water Treatment Calculations - U.S. units					
2. Activated Sludge Operational Calculations					
Instructions: Enter values in blue boxes. Spreadsheet calculates values in yellow boxes					
Inputs			Calculations		
Prim. Effl. Flow Rate, $Q_o$ =	3.5	MGD			
			Recycle Activated		
Prim. Effl. BOD, $S_o$ =	175	mg/L	Sludge Flow Rate, $Q_r$ =	1.3	MGD
Prim. Effl. TSS, $X_o$ =	200	mg/L	Aeration tank vol. $V_{MG}$ =	1.27	MG
Waste/recycle activated			Waste Activated Sludge		
sludge SS conc., $X_w$ =	7,000	mg/L	Flow Rate, $Q_w$ =	0.030	MGD
Aeration tank vol., $V$ =	170,275	ft <sup>3</sup>	Aeration tank <b>F:M</b> =	0.32	
			(lb BOD/day/lb MLVSS)		
Aeration tank MLSS, $X$ =	2000	mg/L			
% volatile MLSS, %Vol =	75%				
Sludge ret. time, SRT =	12	days			

To solve this Example problem, the given values (  $Q_o = 3.5$  MGD,  $S_o = 175$  mg/L,  $X_o = 200$  mg/L,  $X_w = 7000$  mg/L,  $V = 170,275$  ft<sup>3</sup>,  $X = 2000$  mg/L,  $\% \text{ Vol} = 75\%$ ,  $\text{SRT} = 12$  days ) were entered into the blue cells in the upper left portion of the spreadsheet. The spreadsheet then calculated the parameters in the yellow cells in the upper right portion of the first page of the spreadsheet.

As shown in the screenshot above, the same values for  $Q_r$ ,  $Q_w$ , and  $F:M$  are obtained as those shown above for Example #1 and Example #2:

$$Q_r = \underline{1.3 \text{ MGD}}$$

$$Q_w = \underline{0.030 \text{ MGD}}$$

$$F:M = \underline{0.32 \text{ lb BOD/day/lb MLVSS}}$$

Calculations in S.I. units can be made using the following equations:

- $Q_r = [Q_o(X - X_o) - Q_w X_w] / (X_w - X)$  [sometimes approximated by:  
 $Q_r = Q_o(X - X_o) / (X_w - X)$  ] (see discussion in next section of course)
- $Q_w = (V * X) / (\text{SRT} * X_w)$
- $F:M = (S_o * Q_o) / (\% \text{ Vol} * X * V)$

Note that these equations are the same as those used for U.S. units except that there is no need to calculate  $V_{\text{MG}}$ , because volume in m<sup>3</sup> can be used for all of the calculations.

## 10. Explanation of Equations for $Q_w$ and $Q_r$

Most of the equations presented and used above are rather straightforward application of a loading factor, calculation of detention time as volume divided by flow rate, or the equation follows directly from the units. The sources of the equations for  $Q_w$  and  $Q_r$  aren't quite as obvious, however, so they are discussed briefly here.

Waste Activated Sludge Flow Rate: The equation for waste activated sludge flow rate,  $Q_w$ , is based on the principle that the average length of time activated sludge solids stay in the aeration tank [the sludge retention time (SRT) or sludge age] is equal to the mass of solids in the aeration tank divided by the rate at which solids are being wasted from the system. In equation form:

$$\text{SRT} = \text{lb act. sludge in aeration tank} / (\text{lb act. sludge leaving system/day})$$

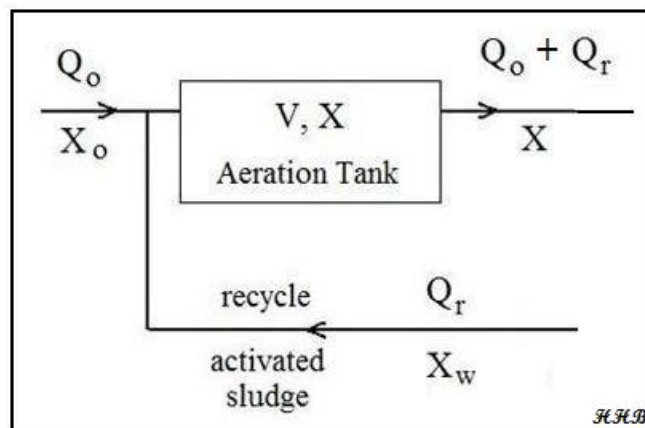
$$\text{SRT} = (8.34 * X * V_{\text{MG}}) / (8.34 * X_w * Q_w) \quad (\text{Note that the factor 8.34 converts mg/L to lb/MG.})$$

$$\begin{aligned} \text{Units in above equation are: } & [(\text{lb/MG}) * \text{MG}] / [(\text{lb/MG}) * \text{MG/day}] \\ & = \text{lb} / (\text{lb/day}) = \text{days} \end{aligned}$$

solving for  $Q_w$  gives the equation in the list above:

$$Q_w = (V_{\text{MG}} * X) / (\text{SRT} * X_w)$$

Recycle Activated Sludge Flow Rate: An equation for the recycle activated flow rate can be determined by a material balance around the aeration tank. The aeration tank portion of the ‘activated sludge parameters’ diagram from above is reproduced on the next page. It shows that the inflows to the aeration tank are  $Q_o$  with suspended solids concentration of  $X_o$  and  $Q_r$  with suspended solids concentration of  $X_w$ . The outflow from the aeration tank is  $Q_o + Q_r$  with suspended solids concentration of  $X$  (equal to that in the aeration tank.)



Aeration Tank Material Balance Diagram

A material balance over the aeration tank must take into account the fact that there is a net growth of activated sludge solids in the aeration tank. The material balance is thus:

Rate of solids outflow – Rate of solids inflow = Growth rate of solids

The growth of activated sludge is typically hydraulically controlled with the activated sludge wasting rate and is equal to  $Q_w X_w$ . The material balance equation thus becomes:

$$(Q_o + Q_r)X - (Q_o X_o + Q_r X_w) = Q_w X_w$$

For S.I. units, the flow rates in the above equation will be in  $m^3/\text{day}$  and the suspended solids concentrations will be in  $\text{kg}/m^3$ , thus giving units of  $\text{kg}/\text{day}$  for each term in the equation.

For U.S. units, the flow rates in the equation will be in MGD and the suspended solids concentrations will be in  $\text{mg}/\text{L}$ . A factor of 8.34 will be needed with each term to convert  $\text{mg}/\text{L}$  to  $\text{lb}/\text{MG}$ , resulting in  $\text{lb}/\text{day}$  for each term. Since I am going to solve the equation for  $Q_r$ , the 8.34 in each term will ‘cancel out.’

Through a bit of algebraic manipulation, the equation can be solved for  $Q_r$  to give:

$$Q_r = [Q_o(X - X_o) - Q_w X_w] / (X_w - X)$$

The activated sludge wasting rate,  $Q_w$ , is typically much less than the influent flow rate,  $Q_o$ , so the term  $Q_w X_w$  is sometimes dropped out to simplify the equation to:

$$Q_r = Q_o(X - X_o) / (X_w - X)$$

## 11. Summary

Extended aeration, contact stabilization, completely mixed activated sludge, and conventional activated sludge are variations of the widely used activated sludge process for biological wastewater treatment. Flow diagrams for these processes were presented along with the parameters and equations used for activated sludge aeration tank design and operational calculations in both U.S. and S.I. units. Example solutions using an Excel spreadsheet were also presented and discussed.

## 12. References

1. Metcalf & Eddy, Inc, (revised by Tchobanoglous, G, Burton, F.L., Stensel, H.D., *Wastewater Engineering Treatment and Reuse*, 4th Edition, New York, NY, 2003.